Midterm #2 August 9, 2004	**American University of Beirut** **Faculty of Engineering and Architecture** **Electrical and Computer Engineering Department** **Signals and Systems EECE 440** **Summer 2004** **Solution**

Problem 1 (2 pts)

Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

a. $x(t) = \cos\left(4t + \frac{\pi}{3}\right)$. Periodic with a period T = π/2 sec.

b. $x(t) = e^{j(\pi t - 1)}$. Periodic with a period T = 2 sec.

Problem 2 (3 pts):

Categorize each of the following signals as an energy signal or a power signal. Show your work.

a. The continuous-time signal x(t), defined by:

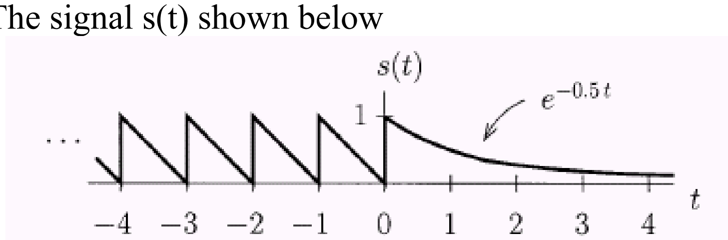
$$x(t) = \begin{cases} 3e^{-2t} & t \geq 0 \\ 0 & \text{Otherwise} \end{cases}$$

Energy signal; $\int_0^{\infty} 9e^{-4t} dt = 9/4$

b. The continuous-time signal y(t), defined for $-\infty < t < \infty$ by
$y(t) = 3\sin(\pi t) + 2\cos(3\pi t)$

Power signal as y(t) is a periodic signal

c. The signal s(t) shown below



None

Problem 3 (4 pts)
The input-output relationship of a continuous-time system is given by:

$$y(t) = x(t/2)$$

a. Is this system linear? Show your work.
 Linear
b. Is this system time-invariant? Show your work.
 Not time-invariant
c. Is this system causal? Show your work.
 Not causal y(-1)=x(-0.5)
d. Is this system stable? Show your work.
 Stable; x(t) bounded implies x(t/2) bounded

Problem 4 (4 pts)
The input-output relationship of a continuous-time system is given by:

$$y(t) = \int_{-\infty}^{t-1} 2x(\tau)\,d\tau$$

a. Is this system linear? Show your work.
 Linear (Integration)
b. Is this system time-invariant? Show your work.
 Time-invariant
c. Is this system causal? Show your work.
 Causal, h(t)=2u(t-1)
d. Is this system stable? Show your work.
 Not stable; H(s) has a pole at s=0

Problem 5 (2 pts)
The impulse response for a linear, time-invariant system is given by:

$$h(t) = ta^{-at^2}, \qquad a>0.$$

Let the input to the system be x(t) = u(t+2) – u(t-2).

a. Is the system stable? **Show work!**
 Use Matlab
b. Find the output of the system.

Problem 6 (2 pts)

When an impulse δ(t) is applied to a certain linear system, the output is $e^{-4t}u(t)$.

a. What output results from applying the input $e^{-t}u(t)$?

$$H(s) = \frac{1}{s+4},\ \ X(s) = \frac{1}{s+1};\ \ Y(s) = \frac{1}{(s+4)(s+1)}$$

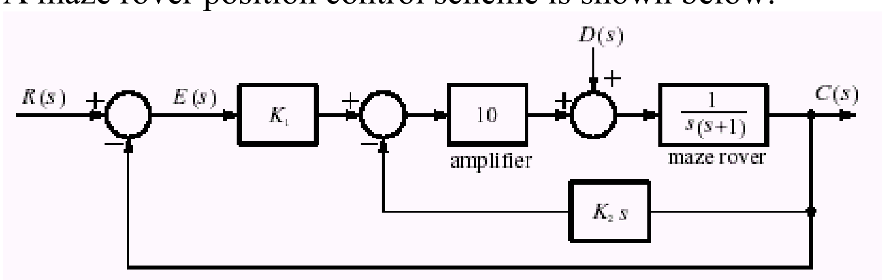
$$y(t) = \left[-\frac{1}{3}e^{-4t} + \frac{1}{3}e^{-t}\right]u(t)$$

b. What output is the result of the input 4 cos(3t + π/4)?

$$y(t) = \left\lfloor Ae^{-4t} + Be^{-3jt} + Ce^{3jt} \right\rfloor u(t)$$

Problem 7 (4 pts)

A maze rover position control scheme is shown below:



a. Determine the transfer functions

1. $$\frac{C(s)}{R(s)} = \frac{10K_1}{s^2 + (1 + 10K_2 s) + 10K_1}$$

2. $$\frac{C(s)}{D(s)} = \frac{1}{s^2 + (1 + 10K_2 s) + 10K_1}$$

b. Determine the values for the constants K_1 and K_2, to obtain a system damping ratio of ζ=0.5 and a 1% steady-state error for a disturbance unit-step, d(t)=u(t). [**NOTE**: Ignore the reference input]

c. What is the steady-state error resulting from the application of a unit-ramp reference input, r(t)=tu(t), with K_1=10 and K_2=0.9? [**NOTE**: Ignore the disturbance input]

Problem 8 (4 pts)

The input-output relationship of a Discrete-time system is given by:

$$y[n] = \sum_{k=-\infty}^{n} x[k+1]$$

a. Is this system linear? Show your work.
   Linear
b. Is this system time-invariant? Show your work.
   Time-invariant
c. Is this system causal? Show your work.
   Not causal (Future values)
d. Is this system stable? Show your work.
   Not stable; y[n] not bounded

Problem 9 (4 pts)

The input-output relationship of a Discrete-time system is given by:

$$y[n] = x[n] \sum_{k=-\infty}^{n} \delta[n-3k]$$

a. Is this system linear? Show your work.
   Linear
b. Is this system time-invariant? Show your work.
   Not time-invariant
c. Is this system causal? Show your work.
   Causal
d. Is this system stable? Show your work.
   Stable

Problem 10 (3 pts)

Assume that the response of an LTI system to the input x[n]=u[n] (discrete-time unit step) is given by,

$$y[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$$

a. For this system, compute the output $y_2[n]$ to the following input:
   $x_2[n] = 3u[n] - 2u[n-4]$
   $$y_2[n] = 3\delta[n] + 6\delta[n-1] - 3\delta[n-2] - 2\delta[n-4] - 4\delta[n-5] + 2\delta[n-6]$$
b. Derive the impulse response h[n] for this system.
   $$h[n] = \delta[n] + \delta[n-1] - 3\delta[n-2] + \delta[n-3]$$
c. Give the difference equation for this system.
   $$y[n] = x[n] + x[n-1] - 3x[n-2] + x[n-3]$$

Problem 11 (4 pts)

Given an IIR filter defined by the difference equation:

$y[n] = \sqrt{2}\, y[n-1] - y[n-2] + x[n]$

a. Determine the transfer function H(z) for this system.

$$H(z) = \frac{1}{1 - \sqrt{2}z^{-1} + z^{-2}}$$

b. Compute the system poles.

$z_1 = e^{j\pi/4}$ and $z_2 = e^{-j\pi/4}$

c. Compute h[n] for this system. Is this system BIBO-stable?

$h[n] = [A\cos(\pi n/4) + B\sin(\pi n/4)]u[n]$

d. Determine y[n] for **x[n] = δ[n] - 3δ[n-1] + 2δ[n-4]**

$y[n] = [\cos(\pi n/4) + \sin(\pi n/4) - 3\sqrt{2}\sin(n\pi/4)]u[n]$

Problem 12 (2 pts)

Determine the discrete-time signals $x_a[n]$ and $x_b[n]$, respectively, corresponding to the following z-transforms:

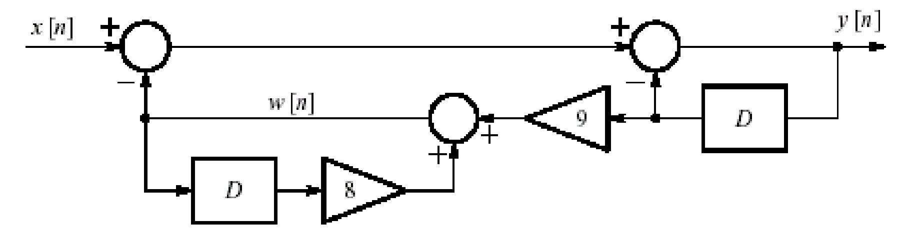
$$X_a(z) = \frac{1 - z^{-1}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

$$x_a[n] = \left\{\frac{8}{5}\left(-\frac{1}{3}\right)^n - \frac{3}{5}\left(\frac{1}{2}\right)^n\right\}u[n]$$

$$X_b(z) = \frac{1 + z^{-1}}{1 - 0.1z^{-1} - 0.72z^{-2}}$$

$$x_b[n] = \left\{\frac{19}{17}(0.9)^n - \frac{2}{17}(-0.8)^n\right\}u[n]$$

Problem 13 (2 pts)
A linear time-invariant discrete-time system is given by the figure shown:



a. Determine the transfer function of the system.

$$H(z) = \frac{1 - 8z^{-1}}{(1 - 2z^{-1})(1 + 4z^{-1})}$$

b. Determine the unit-step response of the system. (1 pt)

$$y[n] = \{A(2)^n + B(-4)^n + C(-1)^n\} u[n]$$

Problem 14 (4 pts)
Given the *Z*-transform

$$X(z) = \frac{2z^2 - \frac{2}{3}z}{z^2 - \frac{7}{2}z + \frac{3}{2}}$$

a. Determine the partial fraction expansion of X(z)/z (1 pt)

$$\frac{X(z)}{z} = \frac{-2/15}{z - 0.5} + \frac{32/15}{z - 3}$$

b. Find the inverse *Z*-transform *x*(*n*) for the following regions of convergence:

i. $|z| > 3$

$$x[n] = -\frac{2}{15}(0.5)^n u[n] + \frac{32}{15}(3)^n u[n]$$

ii. $|z| < ½$

$$x[n] = \frac{2}{15}(0.5)^n u[-n-1] - \frac{32}{15}(3)^n u[-n-1]$$

iii. $1/2 < |z| < 3$

$$x[n] = -\frac{2}{15}(0.5)^n u[n] - \frac{32}{15}(3)^n u[-n-1]$$

Problem 15 (2 pts)

The open-loop transfer function of a unity feedback control system is given by:

$$G(s) = \frac{3s+10}{s^3 + 2s^2 + s}$$

a. Determine the steady-state value (final value) of the output signal for a unit-step input.

$$C(s) = \frac{3s+10}{s(s^3 + 2s^2 + 4s + 10)}$$

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} sC(s) = 1 \quad \text{(System is stable)}$$

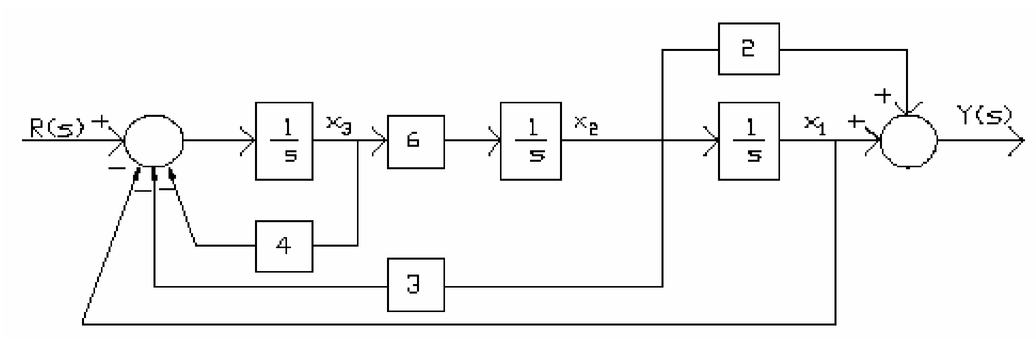
b. Represent this system in a state variable form by determining the state equation and the output equation

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \\ X_3'(t) \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 0 & 1 \\ -10 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \\ 10 \end{bmatrix} r(t)$$

$$c(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix}$$

Problem 16 (2 pts)

Given the block diagram below, write the state equations for this system



$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \\ X_3'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 6 \\ -1 & -3 & -4 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$c(t) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix}$$

Problem 17 (2 pts)

Apply the Routh-Hurwitz (RH) stability criterion to the following characteristic equations $Q(s) = s^4 + 2ks^3 + 2s^2 + (1+k)s = 0$ and determine the range of *k* for stability

As s=0 is pole, system is unstable for all values of K.